Train Schedule Optimization based on Weighted Objective Planning

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Abstract**—**Urban rail transit is a crucial part of the city's transportation system. Especially during the city's morning and evening peak hours, the surge in passenger numbers puts significant pressure on the operation of the rail system. For operators, designing a reasonable train timetable is an effective way to reduce operating costs while improving service quality.The optimization of train timetables is a classic issue in the field of urban rail transportation. Typically, this problem is addressed using linear or nonlinear programming techniques. Due to the diversity of objectives and the complexity of constraints, professional computing software is often required to complete the optimization quickly. Among many mathematical programming software options, Gurobi is favored for solving large-scale mathematical planning problems due to its simple user interface, fast computational capabilities, detailed documentation support, and a free policy for the academic community. In this study, Gurobi was chosen as the main programming tool to build a mathematical model based on detailed data provided and to complete the optimization using its Python interface, successfully solving the following two issues.

First, with the assurance of meeting passenger demand, the objective is to minimize the company's operational costs and maximize service levels, leading to the formulation of a train operation plan. This specifically includes determining the number of trains for the main operational segments and the operating intervals and number of trains for the secondary segments.Train operations employ a 1:n or n:1 loop mode, with the goal of minimizing the company's operational costs (including the number of trains and the mileage) and the passengers' waiting costs (including time on the train and waiting time), converting these costs into economic costs. To achieve this, a weighted objective planning model was established, with a 0.7 weight assigned to the company's operating costs and a 0.3 weight to passengers' waiting time. This model was developed and optimized using Gurobi.To further reduce operational costs and enhance service levels, we conducted sensitivity analysis based on changes in external conditions, proposing methods to improve train operation schemes and schedules.

Keywords**—***Rail operations; Mathematical optimization; Goal programming; Nonlinear programming*

I. INTRODUCTION

In contemporary society, rapid socio-economic development has significantly improved living standards, leading to an increase in the number of urban vehicles, especially private cars, which are growing rapidly. This not only leads to urban traffic congestion but also potentially delays the implementation of national energy-saving and emission reduction plans. To effectively prevent the continuous occurrence of this phenomenon, the government continuously encourages local cities to build subways, which can effectively alleviate urban traffic pressure. Under the national policy of vigorously promoting urban rail transit, the optimization of train timetables in the field of urban rail transit operations is one of the classic problems[1].

The design of train timetables essentially involves setting a conflict-free route for each train while specifying the arrival and departure times at each station. This design process first requires the establishment of a train operation plan, that is, route planning. In planning, it is necessary to consider the number of stations, the ratio of different types of trains, passenger in-vehicle time, waiting time, and section passenger flow among various data factors. Additionally, the handling capacity and service level of the stations are also crucial constraints to consider. Based on ensuring the satisfaction of passenger flow demand, the operation plan should aim to minimize operating costs and maximize service levels, achieving the shortest waiting time and the longest in-vehicle time for passengers. Based on this plan, the specific departure and arrival times of trains are then determined. Therefore, formulating the operation plan is one of the key links in the research of urban rail transit timetable optimization.

In this paper, we set two core objective tasks aimed at deeply analyzing the key challenges in this field. First, considering the continuous growth of passenger flow demand, our first objective task is to design an efficient train operation plan to minimize operating costs and maximize service levels. This includes determining the ideal number of trains within the major loop intervals and the operating intervals and number of trains for the minor loops, ensuring both passenger travel needs and operational cost control are met.

Finally, to comprehensively explore strategies for reducing operational costs and enhancing service quality, the second objective task involves using quantitative analysis methods, based on detailed passenger flow and station data, to propose and evaluate a series of innovative strategies and recommendations. These strategies aim to provide scientific justification and practical guidance for the sustainable development of the urban rail transit system.

II. PROBLEM ANALYSIS

A. *Analysis of Objective One*

In the major and minor loop mode, assuming that the ratio of the number of major and minor loop trains is an integer, information such as OD passenger flow distribution, segment running times, and related train operation parameters can be used to determine the train operation ratio and turn-back stations for major and minor loop sections. Under the constraints of meeting passenger flow requirements, maximum/minimum departure time intervals, and minimum train tracking interval, the goal is to maximize service levels and minimize corporate operating costs. A bi-objective mixed-integer nonlinear programming model is constructed for the urban rail transit operation plan.

B. Analysis of Objective Two

Starting from the solution to Objective One, we conducted a cost analysis on the proportion of passenger flow within the minor loop starting stations and sections compared to the major loop sections. By continuously adjusting the proportion of passenger flow in the minor loop, i.e., changing the ratio of minor loop passenger flow relative to major loop passenger flow, we studied the specific impact of this proportion change on costs. Additionally, we examined the impact of these proportion adjustments on costs during different periods of actual train operation.

III. MODEL ASSUMPTIONS

To facilitate addressing the problem without compromising model accuracy, the following assumptions are made:

- Assume a uniform train type and configuration for train compositions.
- Assume that trains stop at every station, regardless of whether the station is a junction.
- Assume that urban rail transit operates in a unidirectional manner.
- Assume passengers board and alight simultaneously, with train dwell times determined by boarding and alighting times.
- Assume urban rail transit operates on a two-level loop system; the more loops, the more complex the operation.
- Consider only two key factors: the operating costs and service levels of the enterprise.
- Assume that major and minor loop trains operate independently, without affecting each other, even within the minor loop sections.
- Assume all trains are unaffected by major or minor loop sections and that each train runs at the same speed.
- Assume passengers arrive at stations evenly and do not consider situations where passengers are delayed.
- Assume that stations with turn-back capabilities have equal turn-back operation times.
- Assume no consideration of variations in passenger flow during peak morning and evening hours.
- Assume passengers opt for direct trains to reach their destinations.

IV. DESCRIPTION OF SYMBOLS

Symbol	*Description*	*Unit*
sm_k	Station spacing k={1,2, ... ,29}	Km
ω_1	Weighting of maximum service level	
ω_2	Weighting of enterprise operating costs	
c_1	Passenger unit time costs	yuan/s
c_2	Operating distance cost	yuan/km
T_{init}	Runtime period, 3600 seconds	s
T_{wait}	Passenger waiting time	
$D_{k,j}$	OD passengers boarding at station k and alighting at station j	
D_n	All OD traffic from station n	
t_{run}	Train pure running time	s
$t_{trace} = 108$	Minimum Trace Interval 108s	s
t_op_i	The interval operation time, i={1,2, ... ,29} denotes the interval	
M_i	The cross-section passenger flow i={1,2, ... ,29}	

V. MODELING AND SOLUTION

A. Design of the urban rail running program

1) Abstract description of the operation mode of large and small interchanges

In a unidirectional train operation model, assuming there are N stations, with a schematic diagram of the major and minor routes as shown in Fig.1, the set of stations is represented as$\{s_j | j = 1,2,3,\dots,N\}$. Major route trains start from station 1 and run through to station N, covering the entire line, whereas minor route trains operate between stations s_a and s_b, and run exclusively in areas with higher passenger volumes in order to alleviate the operational pressure on the major route trains during peak passenger times.

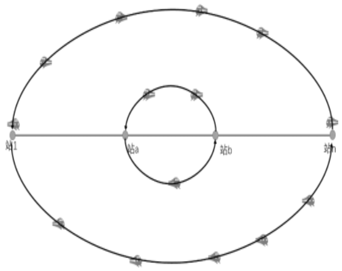

Fig. 1. Schematic Diagram of Major and Minor Route Trains

During a specific operational period, the passenger flow from station $s_k$boarding and alighting at station s_j is denoted as D_{kj}. The total passenger flow departing from station s_k and arriving at each station in the interval $(s_k \text{to} N)$ is given by $D_k = \sum_{j=k+1}^{N} D_{kj}$. Assuming the ratio of the number of major route trains to minor route trains is $1{:}\,m$[2], and if the number of major route trains is x, then the number of minor route trains is mx. Based on the origin-destination (OD) passenger data, the OD passenger flow for the major route is denoted as M_1, and for the minor route as M_2. Under the assumption, M_1 can only be carried by major route trains, while M_2can be carried by both major and minor route trains. The sharing ratios for majorand minor route trains are $\theta = \frac{1}{1-m}$ and $1-\theta$, respectively.

In a unidirectional train operation mode, the dataset includes a total of 30 stations, with the minor route segment running from station 9 to station 24. Passenger flows are categorized into six types as illustrated in Fig.2. Type I, II, III, and VI passengers are restricted to using major route trains to reach their destinations. Type IV passengers have the flexibility to travel to their destinations via either major or minor route trains. Type V passengers initially use the minor route train to arrive at the endpoint of the minor route at station 24, then transfer to a major route train to reach their final destination. This categorization is extended to a network of n stations.

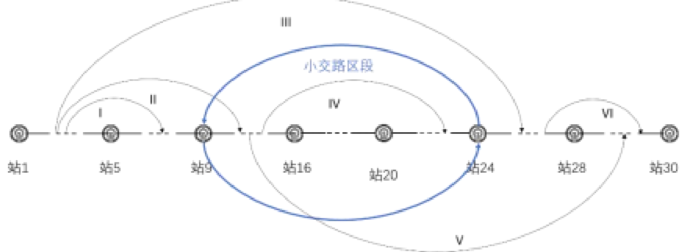

Fig. 2. Passenger Classification Diagram.

2) Modeling

In addressing Objective One in the formulation of operational plans, two key aspects must be considered:

corporate costs and service levels. The operational plan model for major and minor route trains aims to minimize operational costs while maximizing service levels. Corporate operating costs include fixed costs, related to the number of vehicles required, and variable costs, associated with the total kilometers traveled by the trains. Service levels are determined by two factors: the time passengers spend on the train and their waiting time at stations. The overall service quality canbe assessed by the total travel time of passengers, which consists of the trains' pure travel time and the waiting time at each stop. It follows that shorter station stops and reduced waiting times enhance service levels, and vice versa. The framework for the specific operational plan is illustrated in Fig.3.

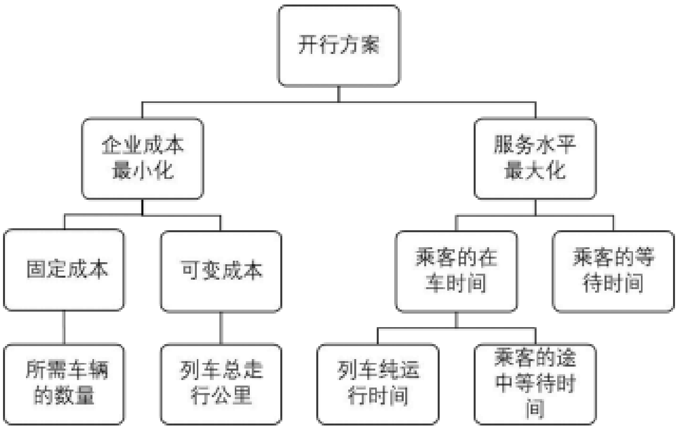

Fig. 3. Operational Plan Design Framework.

Based on the cross-sectional passenger flow data, which provides the passenger volume per unit of time for each cross-section, and considering the train operating times between intervals, it's evident that the intervals correspond to these cross-sections, allowing the use of each interval's running time as the operational time for each cross-section. With the running times for each interval provided, we can calculate the total in-train passenger time across the 30 stations and 29 intervals. Additionally, by incorporating the passenger flow of each cross-section, we can estimate the number of train operations for each cross-section.

The table below assigns identification numbers to each cross-section as shown in Tab. 1.

TABLE I. CROSS-SECTION NUMBERING

Cross-section Numbering	section	Number of Train Runs per Cross-section
1	Station 1->Station 2	2
2	Station 2->Station 3	4
3	Station 3->Station 4	4
4	Station 4->Station 5	7
5	Station 5->Station 6	9
6	Station 6->Station 7	12
7	Station 7->Station 8	17
8	Station 8->Station 9	17
9	Station 9->Station 10	19
10	Station 10->Station 11	20
11	Station 11->Station 12	25
12	Station 12->Station 13	26
13	Station 13->Station 14	25
14	Station 14->Station 15	25
15	Station 15->Station 16	19
16	Station 16->Station 17	18
17	Station 17->Station 18	17
18	Station 18->Station 19	17
19	Station 19->Station 20	17
20	Station 20->Station 21	17
21	Station 21->Station 22	17
22	Station 22->Station 23	17
23	Station 23->Station 24	6
24	Station 24->Station 25	6
25	Station 25->Station 26	6
26	Station 26->Station 27	5
27	Station 27->Station 28	5
28	Station 28->Station 29	4
29	Station 29->Station 30	3

The curve showing the cross-sectional passenger flow is illustrated in Fig.4:

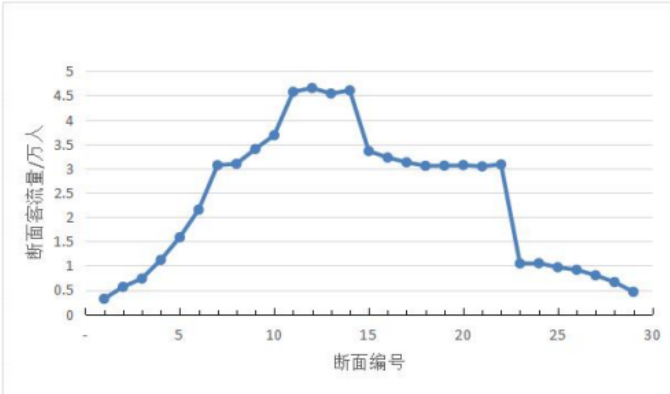

Fig. 4. Cross-sectional Passenger Flow Curve

*a) Passenger In-Vehicle Time:*From the preceding analysis, passenger in-vehicle time is divided into two segments: the train's pure travel time and the passengers' waiting time during transit. Consideration is given to each aspect as follows:

1. Train Pure Running Time:

From the analysis of OD data, it is known that $D_{k,j}$ represents the number of passengers from station s_k to station s_j, corresponding to the data in the cell of the OD table. We need to calculate the train running time from station s_k to station s_j, denoted as $R_{k,j}$. The calculated $R_{k,j}$ times are presented, and due to the large size of the table, only a part is excerpted, as shown in Tab. 2.

TABLE II. PURE OPERATING TIME BETWEEN ANY TWO STATIONS

	Station 1	Station 2	Station 3	Station 4	Station 5
Station 1	0	120	217	318	407
Station 2	0	0	97	198	287
Station 3	0	0	0	101	190
Station 4	0	0	0	0	89
Station 5	0	0	0	0	0

After calculating Rk,j, the total pure running time of the train can be determined. The formula is expressed as follows:

$$t_{run} = \sum_{k=1}^{30}\sum_{j=1}^{30}(D_{k,j} \times R_{k,j}) \quad (1)$$

2.Passengers' Waiting Time En Route:

The mid-journey waiting time for passengers essentially refers to the sum of the train's dwell times at each station between boarding and alighting points. The complexity of calculating waiting times increases due to varying passenger flow patterns on different segments of the route and the differing frequencies of major and minor route trains across these segments.

From the previous analysis, passengers are categorized into six types, and the waiting time en route is calculated individually for each type. For types I, II, and III, who exclusively travel on major route trains and do not transfer mid-journey, the formula for calculating their waiting time is as follows:

First, calculate the sum of all OD passenger flows boarding at station n, denoted as Dn, with the specific formula as follows:

$$D_n = \sum_{j=n+1}^{30} D_{n,j} \quad (2)$$

During a given operating period, the average en-route waiting time for passengers at station n equals the average boarding time multiplied by the total passenger flow boarding at station n, D_n, divided by the number of major route trains, x.

$$t_{sd1} = 0.04 \times \left[\sum_{k=1}^{a-1}\sum_{j=k+1}^{30}\left(D_{k,j} \times \sum_{n=k}^{j-1} D_n \times \frac{1}{x}\right)\right] \quad (3)$$

Here, "a" represents the starting station of the minor route, while "k" and "j" indicate the station numbers where passengers board and alight, respectively. $D_{k,j}$ is the OD passenger flow from station k to station j. The value 0.04 represents the average boarding time for passengers.

Since Type IV passengers can only travel on minor route trains, the formula for calculating their waiting time en route is as follows:

$$t_{sd4} = 0.04 \times \left[\sum_{k=a}^{b-1}\sum_{j=k+1}^{b}\left(D_{k,j} \times \sum_{n=k}^{j-1} D_n \times \frac{1}{x+mx}\right)\right] \quad (4)$$

In areas where major and minor route trains overlap, the number of trains operating includes both major and minor route trains, represented as x + mx, where b is the endpoint of the minor route.

Type V passengers may travel directly to their destination on major route trains or may first take a minor route train to its endpoint and then transfer to a major route train. The formula for calculating their waiting time en route is as follows:

$$t_{sd5} = 0.04 \times \sum_{k=a}^{b-1}\sum_{j=n+1}^{30}\left[D_{k,j} \times \left(\sum_{n=k}^{b-1}\sum_{q=b+1}^{30} D_{n,q} \times\right.\right.$$
$$1x+mx+n=kj-1Dn\times1x \quad (5)$$

Type VI passengers can only travel on major route trains, and the specific calculation formula is as follows:

$$t_{sd6} = 0.04 \times \left[\sum_{k=0}^{30}\sum_{j=k+1}^{30}\left(D_{k,j} \times \sum_{n=k}^{j-1} D_n \times \frac{1}{x}\right)\right] \quad (6)$$

The total waiting time of passengers in the diagram is denoted as T_{sd}, and the calculation formula is as follows:

$$T_{sd} = t_{sd1} + t_{sd4} + t_{sd5} + t_{sd6} \quad (7)$$

Therefore, the in-vehicle time for passengers is denoted as T_{int}, and the calculation formula is as follows:

$$T_{intsd} = t_{run} + T_{sd} \quad (8)$$

Passenger Waiting Time: Based on the assumption that passengers arrive at stations uniformly, and given the shorter intervals between train operations, the average waiting time for passengers in both major and minor route sections is approximately half the interval between trains[3]. Therefore, the average waiting time for passengers in the major route section is $\frac{T_{init}}{2x}$, (with the data indicating train operations from 7:00 AM to 8:00 AM, which is 3600 seconds). The average waiting time for passengers on minor route trains is denoted as $\frac{T_{init}}{2(1+m)x}$[4].

Passenger Waiting Time = Waiting Time in Minor Route Section + Waiting Time in Major Route Section.

Calculation of Waiting Time for Passengers on Minor Route Trains:

The passenger flow covered by the minor route section is denoted as PA_1.

$$PA_1 = \sum_{k=a}^{b-1}\sum_{j=k+1}^{b} D_{k,j} \quad (9)$$

The waiting time for passengers in the minor loop section is denoted as t_{w1}, and the calculation formula is as follows:

$$t_{w1} = \frac{T}{2(1+m)x} \times PA_1 \quad (10)$$

Calculation of passenger waiting time for major loop trains:

The passenger flow covered by the major loop section is denoted as PA2.

$$PA_2 = \sum_{k=1}^{30}\sum_{j=1}^{30} D_{k,j} - PA_1 \quad (11)$$

The waiting time for passengers in the major loop section is denoted as t_{w2}, and the calculation formula is as follows:

$$t_{w2} = \frac{T}{2x} \times PA_2 \quad (12)$$

Passenger waiting time is denoted as Twait, and the calculation formula is as follows:

$$t_{wait} = t_{w1} + t_{w2} \quad (13)$$

Fixed costs (number of vehicles required):

The fixed operating costs of the enterprise are denoted as co1, and the calculation formula is as follows:

$$co_1 = x + mx \quad (14)$$

Variable costs (total train mileage): The variable operating costs of the enterprise are denoted as co_2.

$$co_2 = 40.168x + mx \times \sum_{k=1}^{b-1} sm_k \quad (15)$$

Where sm_k represents the operating distance of the minor loop train. Establish the optimization objective function:

Given that the enterprise's costs include fixed costs (i.e., the number of vehicles required) and variable costs (i.e., the total mileage of trains), along with considerations of maximizing service levels by minimizing the time passengers spend on the vehicle and waiting, these two objective functions have different dimensions and units, making it impossible to solve them directly through weighted coefficients. Therefore, after consulting relevant materials, it was decided to convert the passenger time cost and enterprise cost into monetary costs, to facilitate weighted calculations. The weight coefficients are determined as follows after comprehensive consideration:

Where w_1=0.3and w_2= 0.7 represent the weight coefficients, C_1= 0.025 yuan/person·s[5] represents the value of per unit waiting time for passengers, and C_2= 48 yuan/km[5] represents the operating cost per train.

$$\mathrm{minZ} = \omega_1 \times c_1(t_{int} + T_{wait}) + \omega_2[3000000co_1 + c_2 \times co2 \quad (16)$$

$$\left\{\begin{array}{ll}
mx + x \le \frac{t_{int}}{t_{trace}} & i \le a-1, i \ge b-1, \\
\mathrm{t_{int}} = 3600\mathrm{s}, \mathrm{t_{int}} = 108\mathrm{s} & \\
x > max\left[\frac{M_i}{c}\right] & \\
(1+mx) \ge max\left[\frac{M_i}{c}\right] & a \le i \le b \\
I_0 = \frac{3600}{X} & 120 \le I_0 \le 360 \\
I_0' = \frac{I_0}{1+m} & 120 \le I_0' \le 360 \\
20 \le 0.04 \times \sum_{n=k}^{j-1} D_n \times \frac{1}{x} \le 120 & \begin{array}{c} k=1,2,\dots,a-1 \\ j=k+1,\dots,30 \end{array} \\
20 \le 0.04 \times \sum_{n=k}^{j-1} D_n \times \frac{1}{x+mx} \le 120 & \begin{array}{c} k=1,2,\dots,a-1 \\ j=k+1,\dots,30 \end{array} \\
20 \le 0.04\left[\sum_{n=k}^{b-1}\sum_{q=a+1}^{30} D_{n,q} \times \frac{1}{x+mx} + \sum_{n=b}^{j-1} D_n \times \frac{1}{x}\right] \le 120 \quad \begin{array}{c} k=a,\dots,b \\ j=b+1,\dots,30 \end{array} & \\
20 \le 0.04\left[\sum_{n=k}^{j-1} D_n \times \frac{1}{x}\right] \le 120 & \begin{array}{c} k=b,\dots,30 \\ j=k+1,\dots,30 \end{array}
\end{array}\right. \quad (17)$$

In formula (17), I_0 and I_0' respectively represent the departure intervals of major and minor loop trains. M_i, where i={1,2, ..., 29}, denotes the section number for passenger flow, and c represents the train's capacity at 1860 people per train car.

Input the data to derive the operating costs for different scenarios, as shown in Tab.3. Additionally, provide a cost comparison chart, Fig.5, for the different scenarios.

TABLE III. SOPERATING SCHEMES FOR MAJOR AND MINOR LOOP TRAINS

Operating schemes	Major and Minor Loop Train Ratio	Number of Major Loop Trains	tarting Point of Minor Loop Trains	Ending Point of Minor Loop Trains	Operating Costs
1	0.0	30.0	1	30	94513649.07
2	0.33	21.0	2	27	88362602.61
3	0.5	18.0	2	27	85612868.85
4	1.0	13.0	5	25	82700334.27
5	2.0	9.0	5	25	85853393.14
6	3.0	7.0	5	25	88996570.28

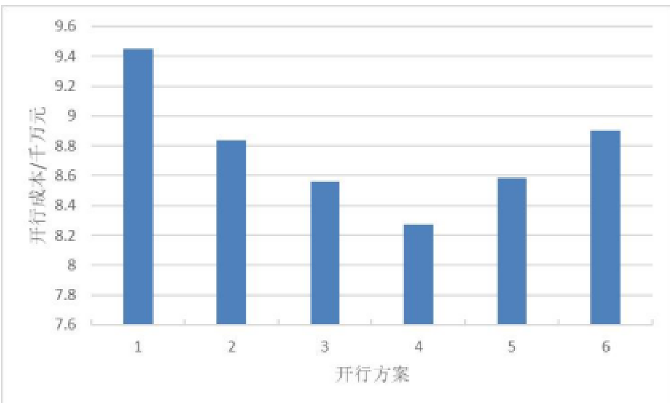

Fig. 5. *Cost Comparison Chart for Operating Schemes*

B. *Solving the model*

Based on the characteristics of Gurobi, such as ease of use, fast computation speed, and broad language support, the model for this problem is built using Gurobi as the optimizer and Python as the programming language for solving the model.

Before modeling, the data is loaded by calling the pandas and numpy libraries to read a simplified Excel spreadsheet. This data includes OD passenger flow, station distances, and available minor loop stations, which are then converted into arrays.

Using the Python interface of the Gurobi optimization software, the model framework M is established. The first step is to convert the unknowns into Python variables. The second step involves breaking down multi-dimensional expressions into simpler one-dimensional and two-dimensional expressions using auxiliary variables to handle complex expressions. The third step is to add constraints for auxiliary variables, intermediate variables, and original constraints sequentially to the model. The fourth step is to set the model's optimization goal in conjunction with the intermediate variables. Finally, the model's optimization function is called to perform the optimization calculations, automatically obtaining the optimal value and model configuration.

After actual execution, the solution time for Problem 1 was 1 minute and 22 seconds. The results output by the model show that the optimal solution costs 82,700,334.27 yuan, ensuring that the total operating costs of the enterprise are minimized while maximizing service levels. From the chart, it can be seen that the optimal minor loop starts at Station 5 and ends at Station 25. The major loop requires 13 trains, and the number of minor loop trains needed matches the major loop in a 1:1 ratio, also totaling 13 trains.

VI. SENSITIVITY ANALYSIS

Based on the design of urban rail service schemes, perform a sensitivity analysis on selecting different minor loop sections. With other parameters held constant, observe the cost variations for different minor loop sections, as shown in Tab.6[6].

TABLE IV. THE IMPACT OF DIFFERENT MINOR LOOP SECTIONS ON COSTS

	Starting Station	Ending Station	Number of Major Loop Trains (x)	Cost	Cost Savings
1	1	30.0	30	94513649.07	0%
2	5	22.0	14	88610794.45	6.25%
3	5	25.0	13	82700334.27	12.50%
4	8	25.0	14	88655326.61	6.19%
5	10	25.0	15	94619205.91	-0.11%

From Tab. 6, we can draw the following conclusions:

In actual train operations, multiple segments for both major and minor loops are available for selection, and decision-makers, often relying on past experiences, usually choose points where section passenger flow suddenly changes as the start stations for minor loops. However, Tab. 6 shows that the start station that minimizes operational costs and maximizes service levels doesn't necessarily coincide with these points of change in passenger flow.

Taking the interval with the highest section passenger flow as an axis and moving the start station of the minor loop forward or backward reveals that including more sections in the minor loop generally reduces the number of major loop trains needed and increases cost savings. This is confirmed by Tab. 6, where moving the starting station of the minor loop from Station 5 to Station 10 actually results in an increase in costs instead of a decrease, with the percentage increase in costs dropping from 12.50% to -0.11%.

As the proportion of passenger flow in the minor loop sections increases, the number of major loop trains operated in the urban rail major and minor loop train mode shows a trend of increase.

As the proportion of passenger flow in the minor loop sections increases, the total cost savings in the urban rail major and minor loop train mode also show a growing trend, indicating that the steeper the curve of the line section passenger flow, the more advantageous it is to implement the major and minor loop mode.

CONCLUSION

First, the model uses weights instead of multiple objectives. It simplifies the problem's objective function, reducing the problem's scale. Unlike multi-objective planning, the weighting method is straightforward, enhances model simplicity, and saves modeling time.

Second, the model addresses constraints thoroughly. For passengers, it not only includes waiting time cost constraints but also in-vehicle time cost constraints. For trains, it includes constraints on the maximum train tracking interval and the maximum section passenger flow, adding constraints for the minimum passenger load.

Third, the model allows for easy adjustment of parameters. For available minor loop stations, it employs an iterative trial approach. By modifying the iteration parameters, the starting and ending points of the minor loop can be quickly determined. The model also uses a ratio of major to minor loop train numbers, m, to establish an integer relationship between the two. By fixing m, it allows flexible changes in the number of trains while maintaining the integer multiple relationship between major and minor loops.

However, the model simplifies the computation by assuming simultaneous boarding and alighting, ignoring the time passengers spend disembarking, which requires more comprehensive calculations in actual use.

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