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Modeling RMB Exchange Rate Volatility – Application of GARCH Family Models

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***Abstract*—**This paper delves into the dynamics of the exchange rate by analyzing 2189 daily median price data of the USD-RMB exchange rate between January 5, 2015 and December 29, 2023. The study adopts the ARMA model to construct the mean equation of the exchange rate, and combines the GARCH family of models, including GARCH, TGARCH and EGARCH, to analyze the data in detail. The analysis results reveal the existence of characteristics such as clustering, asymmetry and leverage effect in the exchange rate of USD-RMB. In general, the GARCH(1,1) model has a more balanced performance, especially in the most important MAPE and MAE indicators, so it can be considered that the GARCH(1,1) model has a better forecasting effect.

Keywords*—RMB exchange rate; ARMA; GARCH family model; leverage effect; asymmetry*

I. INTRODUCTION

The volatility of the USD-RMB exchange rate reflects the instability and risk of market expectations of the RMB exchange rate. Changes in the volatility of the USD-RMB exchange rate are affected by a variety of factors, including domestic and international economic conditions, political events, market sentiment and policy adjustments. The long-term uncertainty in the U.S.-China relationship, the strength of China's domestic economic recovery and changes in the global economic environment are all important influences on the future trend of the RMB exchange rate. In recent years, the volatility of the RMB exchange rate has received widespread attention, especially against the backdrop of increased global economic uncertainty. Under such circumstances, the increased volatility of the RMB exchange rate implies greater exchange rate risk for enterprises and investors and requires corresponding risk management measures.

The objective of this study is to model the volatility of the USD-RMB exchange rate using different time series methods.

II. MATERIALS AND METHODS

The object of empirical research is the median exchange rate of RMB against the US dollar, hereinafter referred to as the RMB exchange rate.In 2015, the central bank reformed the quotation mechanism of the median exchange rate of RMB against the US dollar, which enhanced the close connection between the RMB exchange rate and the exchange rate market, as well as strengthened the role of the exchange rate in the economic and financial fields, but also enhanced the volatility of the RMB exchange rate. Therefore, the data of this paper is selected from the period from the beginning of 2015 to the period of 2023, and the missing data of holidays and individual dates are removed to get a total of 2,189 exchange rate data from the State Administration of Foreign Exchange of China. The raw data are logarithmized and processed with first-order differencing, noting Pt as the daily median price of the USD-RMB exchange rate, Rt as the daily return.

$$R_t = 100 * \ln\left(\frac{P_t}{P_{t-1}}\right) = 100 * (lnP_t - lnP_{t-1}) \quad (1)$$

The ARMA model is a statistical model proposed by George Box and Gwilym Jenkins in 1970. The model incorporates the features of both autoregressive (AR) and moving average (MA) models. In the ARMA model, its smoothness depends mainly on the autoregressive (AR) part, while reversibility is determined by the moving average (MA) part. The autoregressive moving (ARMA) model can be defined.

$$X_t = \varphi_1 X_{t-1} + \cdots + \varphi_{1p} X_{t-p} + e_t - \theta_1 e_{t-1} - \cdots - \theta_q e_{t-q} \quad (2)$$

In 1982, Engle[1]oposed the autoregressive conditional heteroskedasticity (ARCH) model, which is particularly suitable for describing time series data where volatility varies unevenly in speed and magnitude over time.The basic principle of the ARCH model is the assumption that the variance of a time series can be expressed as a linear combination of the squared past errors. On this basis, Engle's student Bollerslev[2]tended the ARCH model in 1986 by proposing the generalized autoregressive conditional heteroskedasticity (GARCH) model.

Mean value equations:

$$\varepsilon_t = \sigma_t Z_t \quad (3)$$

Conditional Variance:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (4)$$

There are several studies in the literature on RMB exchange rate volatility.Wenqian Hang (2023)[3]mparing different distributional assumptions, the EGARCH model under the GED distribution determined by the information criterion is able to match the peak and thick-tailed features of the volatility series very well.Chunyi Lu (2021)[4]e ARMA-GARCH risk forecasting model has a good measurement of market performance, and its accuracy is significantly higher than that of traditional econometric models.Bo Sun (2016)[5]e RMB exchange rate system is a typical nonlinear dynamic and complex system, and the GARCH-type nonlinear structure in the RMB exchange rate series exhibits the characteristics of non-continuity and transience. Introduction to the TGARCH model formulation

The GARCH model can well explain the volatility aggregation characteristic of financial asset return series, but it cannot explain the frequent presence of leverage effects in financial time series. The TGARCH (Threshold GARCH) model is an improved GARCH model that captures the leverage effects in the financial time series, the asymmetry in the response to negative (bad news) and positive (good news) news. The TGARCH model proposed by Zakoian[6] achieves this by introducing a threshold mechanism based on the GARCH model. Its conditional variance is set as follows.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i I_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (5)$$

In order to overcome the nonnegativity constraints on the parameters of the GARCH model, Nelson (1991)[7] proposed an extended form of the GARCH model, the EGARCH model. This model effectively captures the asymmetry in the financial asset return series, the so-called leverage effect, and it relaxes the constraint of non-negativity of the parameters in the GARCH model. Its conditional variance is set as follows.

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2) \quad (6)$$

III. RESULTS

The yield data is plotted as a line graph in Figure 1, from which we can see that the logarithmic yields within the line graph show a clustering phenomenon, higher yields tend to pile up, and lower yields follow lower yields, which is a characteristic of yield volatility that we can call volatility clustering.

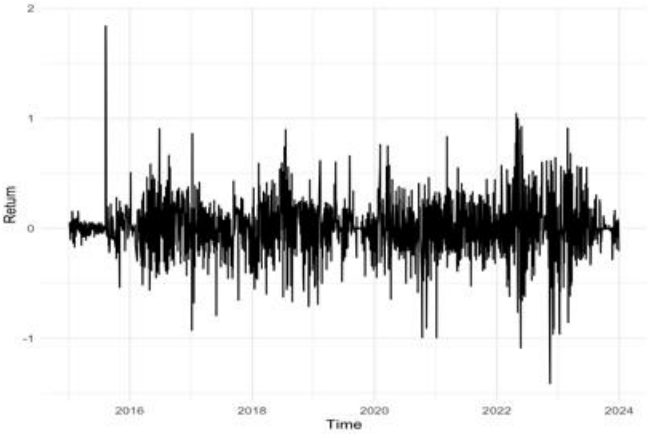

Figure 1: Time series of return

The normality test of the data, as shown in Table 1, reveals that the skewness S is 0.2348361 and the kurtosis K is 8.211239, which reflects that the data show right skewness and spikiness, the logarithmic returns are more likely to be positive and the 2188 sample data have a large variability. Meanwhile, the large value of Jarque-Bera statistic indicates that the sample data do not obey normal distribution.

Table 1: The normality test of the return

sample size	skewness	kurtosis	Jarque-Bera	P-value
2188	0.2348	8.2112	2495.900	0.00

Sample data smoothness is a prerequisite for regression analysis of time series, we will determine the smoothness of a time series by determining whether there is a unit root to determine the smoothness of the series is called the unit root test. In the process of data analysis, in order to avoid the existence of heteroskedasticity in the data, so we choose to use the PP test as a unit root test, the original hypothesis is: H0: $\gamma = 0$, that is, there is a unit root, the sequence is a non-stationary time series. The test obtained a P-value of 0.01 less than 0.05, rejecting the original hypothesis that the series is smooth and can be used to construct the ARMA model. With AIC as the criterion, the automatic fitting model is used, and the ARMA (1, 1) model is finally selected.

$$r_t = 0.9307\, r_{t-1} - 0.8922\, \varepsilon_{t-1} + \varepsilon_t \quad (7)$$

A unit root test is carried out and as shown in the figure, all the roots fall within the unit circle, thus passing the smoothness test, indicating that the established equation is smooth and reversible.

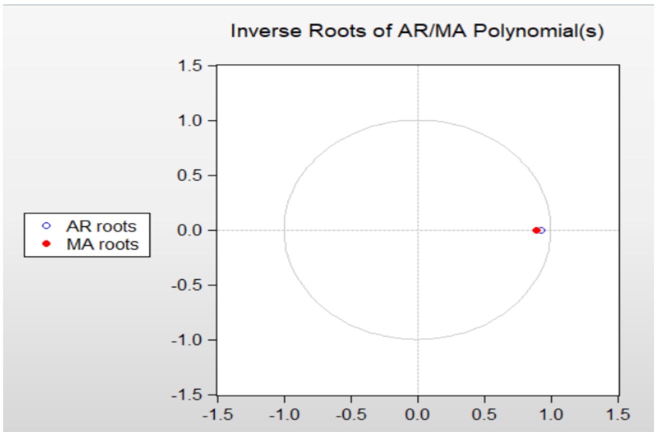

Figure 2: A unit root test

In order to further determine the reasonableness of the modeling, the residual series were then tested to see if the residual series were white noise series, and the Ljung-Box test was used to examine the residual series.

Table 2: The Ljung-Box test

latency order	LB statistic X^2	P-value
20	17.918	0.5928
10	6.4505	0.7761

The p-values of the Ljung-Box statistic at all delay orders exceed the 5% significance level, indicating that there is no reason to reject the hypothesis that the residual series are white noise. This further indicates that the residual series have successfully passed the white noise test, confirming that the mean equation of the ARMA model is adequate in terms of information extraction.

In order to better establish the GARCH model, we need to test the ARCH effect on the data. The least squares method is used for repeated linear regression tests, and the ARMA (1,1) model is used for preliminary fitting of the data, and the coefficients of the resulting optimal mean equations are all significant, and the mean equations are shown below:

$$\gamma_t = 0.66\gamma_{t-1} - 0.57\varepsilon_{t-1} + \varepsilon_t \quad (8)$$

The ARCH-LM test was applied to the residual series using EVIEWS with a lag of 10 and the results obtained from the test are shown in Table 3.

Table 3: The ARCH-LM test

latency order	Chi-squared	p-value
10	271.31	2.2e-16

The LM statistic in Table 3 corresponds to a p-value close to 0, which is less than the significance level of 0.05 and rejects the original hypothesis, thus we can conclude that there is conditional heteroskedasticity in this time series. The results show that the model residual series has a higher order ARCH effect at the 5% significance level, so it is appropriate to use the GARCH model to fit the sample data.

The ARCH test concluded that the data had ARCH effect, and based on the ARMA((1,1) model with mean equation, the GARCH (p,q) model was established to fit the logarithmic data. After repeated fitting and comparison of the experimental data, combined with the statistical significance of the coefficients and the requirements of various aspects, it was found that GARCH (1,1) fits the given data better.

The resulting RMB exchange rate is modeled as follows:

Mean value equations:

$$\gamma_t = 0.9511\gamma_{t-1} - 0.9248\varepsilon_{t-1} + \varepsilon_t \quad (9)$$

Conditional Variance:

$$\sigma_t^2 = 0.0043 + 0.1489\varepsilon_{t-1}^2 + 0.7766\sigma_{t-1}^2 \quad (10)$$

It can be calculated from the formula $\alpha + \beta$ = 0.1489 + 0.7766 = 0.9255, which is close to 1. The GARCH term coefficient of 0.7766 indicates that the volatility of one day's return has an extremely strong correlation to the volatility of the previous day's daily return, which means that the shock to the return has a sustained effect, and that volatility decays more slowly, which will clearly show the phenomenon of volatility aggregation. The ARCH LM test is conducted on the residuals after fitting the GARCH (1,1) model, and the LM statistic corresponds to a P value of 0.9957 which is significantly greater than 0.05, so the original hypothesis is accepted, and the conditional heteroskedasticity has been eliminated, it is considered that there is no ARCH effect in the residuals series.

However, in the process of fitting with GARCH, it is found that the GARCH model does not fit the USD-RMB exchange rate well, and according to the AIC criterion, it is found that the result of applying the GARCH (1,1) fitting is -2.2243, and for this reason, this paper will study other GARCH family models in depth in order to build a more appropriate model.

TGARCH modeling of exchange rate data, ARMA (1, 1) as the mean equation, and the use of Eviews on the data for repeated operation of the experiment to find the TGARCH (3,1) fit better, coefficients of statistical significance are through the test, and the AIC value of -2.993556, the SBC value of -2.942286, the model obtained from the fit is:

Mean value equations:

$$\gamma_t = 0.9539\gamma_{t-1} - 0.9278\varepsilon_{t-1} + \varepsilon_t \quad (11)$$

Conditional Variance:

$$\sigma_t^2 = 0.0032 + 0.1137\varepsilon_{t-1}^2 + 0.1476\varepsilon_{t-1}^2 I_{t-1} - 0.1431\varepsilon_{t-1}^2 I_{t-2} + 0.8272\sigma_{t-1}^2 \quad (12)$$

The results obtained from the ARCH LM test on the residuals after fitting the TGARCH (1,1) model are shown in Table 5:

The results in Table 5 show that the P-values of F and LM statistics are significantly larger than 0.05, so the original hypothesis cannot be rejected, and it is considered that there is no ARCH effect in the residual series, which indicates that the TGARCH (1,1) model can fit the data well enough. The sum of the coefficients of the ARCH and GARCH terms of this equation is less than 1, which means that the impact of information shocks on the USD-RMB exchange rate is relatively short-lived, and the coefficient of the pre-dummy variable is 0.1476, which indicates that the leverage effect exists. ARCH LM test on the residuals, LM statistics corresponding to the P value of 0.9968 is significantly greater than 0.05, so accept the original hypothesis, the condition of heteroskedasticity has been eliminated, that is, the residuals of the series that there is no ARCH effect. Thus, it shows that the TGARCH (1, 1) model can fit the data well.

The EGARCH model was established on the basis of ARMA(1,1), and after repeated fitting and comparison of the experimental data, combined with the statistical significance of the coefficients and the requirements of many aspects, the EGARCH(1,3) was found to be a better fit to the given data. The fitted AIC is -3.026294, SBC is -2.975024 and the fitted model is:

Mean value equations:

$$\gamma_t = 0.9055\gamma_{t-1} - 0.8752\varepsilon_{t-1} + \varepsilon_t \quad (13)$$

Conditional Variance:

$$\ln(\sigma_t^2) = -0.6927 + 0.2884\left|\frac{\mu_{t-1}}{\sqrt{\sigma_{t-1}}}\right| - 0.0305\frac{\mu_{t-1}}{\sqrt{\sigma_{t-1}}} + 0.8389\ln(\sigma_{t-1}^2) \quad (14)$$

The corresponding P-values of F and LM statistics are significantly larger than 0.05, so the original hypothesis is accepted that there is no ARCH effect in the residual series, which indicates that the EGARCH (1, 1) model can fit the data well enough. In addition, in the fitting process, the coefficient of leverage effect in the conditional variance equation is not zero, that is, the information has an asymmetric role, and the volatility of the USD-CNY exchange rate reacts more strongly to good and bad news than good news. The ARCH LM test is performed on the residuals, and the LM statistic corresponds to a P-value of 0.9877 which is significantly larger than 0.05, so the original hypothesis is accepted, and the conditional heteroskedasticity has been eliminated, it is considered that there is no ARCH effect in the residual series.It shows that the EGARCH (1, 1) model fits the data well.

The three models were used separately to predict the data for the next 5 days and only the last three months are shown, the results are shown below .

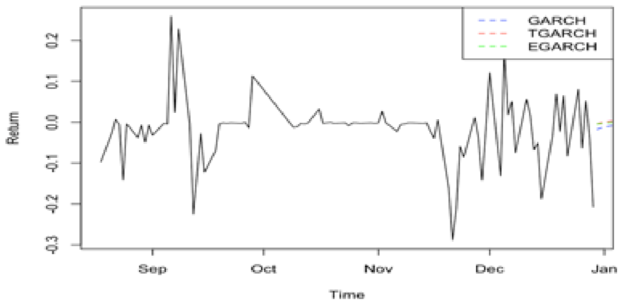

Figure 3: Forecasting with GARCH,TGARCH and EGARCH

Comparison of the prediction results with the true values yields a prediction evaluation, and the prediction effect of each model is shown in the table below.

Table 4: The prediction effect of each model

modle	ME	RMSE	MAE	MPE	MAPE
GARCH	0.0628	0.1576	0.0976	62.1429	89.4463
TGARCH	0.0507	0.1531	0.0982	100.4241	100.4241
EGARCH	0.0527	0.1537	0.0979	93.1078	93.1078

The GARCH(1,1) model performs best in terms of MAE, MPE and MAPE, which implies that it has relatively low and stable forecast errors.The TGARCH(1,1) model performs best in terms of ME and RMSE, which suggests that it has a slight predominance in terms of bias and overall error. In general, the GARCH(1,1) model has a more balanced performance, especially in the most important MAPE and MAE indicators, so it can be considered that the GARCH(1,1) model has a better forecasting effect.

CONCLUSION

In summary, this paper selects the GARCH model, EGARCH model and TGARCH model in the GARCH model family combined with ARMA as the mean equation model to process and fit the data for the daily mid-price of the US dollar to RMB exchange rate, and in addition selects the normal (Gaussian) distribution to establish the time-series model by integrating the models of the GARCH model family, and in the process of the model fitting, we find that the fluctuation of the US dollar to RMB exchange rate has a cluster and asymmetric effect. In the process of model fitting, we find that the fluctuation of USD-RMB exchange rate is clustered and asymmetric, and we also find that the RMB exchange rate has a leverage effect on good news and bad news from the fitting of TGARCH model and EGARCH model. Based on the data of MAPE and MAE indicators, we believe that the GARCH(1,1) model is more effective in forecasting.

In the fitting process we found that since China began to implement the new exchange rate policy, the statistical distribution characteristics of the RMB exchange rate has undergone a series of changes, and its mean value is gradually reduced compared with the previous, and the peak value is much larger than the peak value of the normal distribution3, which indicates that the dollar exchange rate against the RMB does not obey the normal distribution. In addition, the skewness of the distribution of the RMB exchange rate is less than zero, which indicates that the USD-RMB exchange rate exhibits a left thick tail, and from the volatility scatter plot, it can be seen that the logarithmic return exhibits the characteristics of the emergence of high-return clusters, which suggests that the financial time series exhibits the volatility clustering. Through the exchange rate autocorrelation biased autocorrelation diagram, it is concluded that the USD-RMB exchange rate presents a weak correlation with a long-term trailing characteristic, that is to say, it indicates that the exchange rate market has a persistent impact on the information shock.

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