The Mathematical Model of Doppler Frequency Shift in Leo At Ku, K and Ka Frequency Bands

[1]Sidiku Mosunmola B, [2]Dr A.O Agboola, [3]Dr Ale Felix and [4]Abdullahi Mohammed,
[1,2,3]National Space Research & Development Agency, Abuja, Nigeria
[4]Department of Electrical and Computer Engineering Ahmadu Bello University, Zaria, Nigeria

Abstract: The Doppler frequency shift poses the problem of receiving higher or lower frequencies than the original transmitted frequency and it is more severe in the Low Earth Orbit (LEO). This paper presents the mathematical modeling of Doppler frequency shift in LEO at Ku, K and Ka frequency band. The results obtained show that Doppler effect is much more severe in LEO than both in MEO and GEO and having established the effect of Doppler frequency shift in LEO, its variation with respect to the relative distance between satellite and the Earth's terminal for different elevation angles (0^0 - 90^0) at Ku, K, and Ka bands was also investigated. Knowing the new position of the Earth's terminal, the new relative distance between the satellite and the Earth's station was calculated.

I. INTRODUCTION

In recent years, low Earth Orbit (LEO) satellites have been employed to carry signals for large population of simultaneous users in mobile satellite communication systems over communication link. LEO satellites have very wide scientific applications such as but not limited to; remote sensing of oceans, analyses of Earth's climate change, Earth's imagery with high resolution and astronomical purposes. Low earth orbit satellites are also used for data relay and navigation as well as low-cost store-and-forward communications systems. The low earth orbit is located at a height range of 106-2000km above the earth surface (Qingchong, 1999). The speed of LEO satellites is high, with a very negligible round trip delay of about 10-20 ms and its orbital period is about 100 minutes (Raymond, 1996). Each LEO satellite is only visible from the earth for about 10 to 20 minutes. The advantages of Low earth Orbit (LEO) include small propagation loss which allows mobile users to use handsets for direct communication, and a small propagation delay (about 10ms compared to 250ms for Geostationary satellite (GEOS)) for better voice performance, and other interactive services. The Major disadvantages of the LEO orbit are; more satellites are required, there is increased probability of atmospheric drag, phase error and Doppler frequency shift. The reception of higher or lower signal frequencies than the original transmitted frequency is due to phenomenon known as Doppler frequency shift; and it is a practical problem in satellite communication (Levano, 1988). As the satellite moves, a Doppler shift of the downlink is produced as observed by the user terminal and a corresponding frequency shift of the uplink as perceived by the satellite (Gataullin et al., 2010). Notably, satellite in non-geostationary orbit changes its position relative to the Earth. As a result of this, signal experiences a Doppler shift whose value and drift rate are very significant as oppose to that obtained with satellite in geostationary orbit (Naser et al., 2001)

The concept of Doppler frequency shift is applicable to the land mobile radio, including digital cellular transmission link. Here, the cause of Doppler effect could be due to the movement of a mobile unit or natural and constructed obstacles. Natural calamities like torrential rains, raging storms, heavy snowfall e.t.c also cause significant Doppler effect in wireless communication (Kausik et. al, 2007). This Doppler effect can cause failure of radio link. (Jamalipour, 1997).

II. MATHEMATICAL MODELING

Doppler frequency shift could be best computed from the geometry of the satellite dynamic in orbit. In order to accurately calculate the Doppler frequency shift, a relative velocity between satellite and ground terminal is required. Figure 2.1 shows a typical representation of a satellite dynamics in orbit.

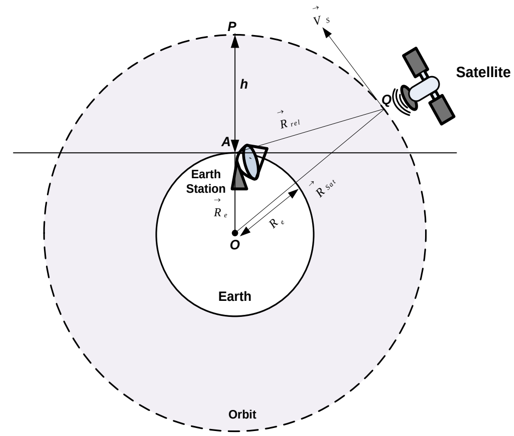

Figure 2.1: Typical Representation of Satellite Dynamics in an Orbit (Naser et al., 2001)

To compute the relative velocity of the Earth's terminal and the satellite, the ground station and the satellite velocity vectors must be in the same coordinate reference frame (Naser et al., 2001). If $\vec{R}_e$ and $\vec{R}_{Sat}$ are the position vectors of the user terminal or Earth's station terminal and satellite respectively, then relative position of the satellite and Eath's terminal is given by (You et al., 2000):

$$\vec{R}_{rel} = \vec{R}_{Sat} - \vec{R}_e \qquad (2.1)$$

The relative velocity of the satellite and Earth's terminal is given by;

$$\vec{V}_{rel} = \frac{d\vec{R}_{rel}}{dt} = \frac{d\left(\vec{R}_{Sat} - \vec{R}_e\right)}{dt} \qquad (2.2)$$

Equation (2.2) can further be expressed in terms of magnitude and direction as;

$$\vec{V}_{rel} = \vec{V}_S \cdot \hat{\vec{R}}_{rel} \qquad (2.3)$$

Where $\hat{\vec{R}}_{rel}$ is the position vector of $\vec{R}_{rel}$ given by;

$$\hat{\vec{R}}_{rel} = \frac{\vec{R}_{rel}}{\left|\vec{R}_{rel}\right|} \qquad (2.4)$$

During Doppler effect, the received and transmitted signal frequencies are related by the following equation (Gataullin et al., 2010);

$$f_r = f_t \pm f_{ds} = f_t \pm \frac{\left|\vec{V}_{rel}\right| f_c}{C} \qquad (2.5)$$

Where:

f_r and f_t are the received and transmitted signal frequencies respectively.

f_c is the carrier wave frequency.

f_{ds} is the Doppler shift frequency.

$\left|\vec{V}_{rel}\right|$ is the magnitude of the relative velocity between the satellite and the Earth's terminal.

C is the velocity of electromagnetic wave.

It can be deduced from equation (2.5) that the Doppler shift frequency is given by (Gataullin et al., 2010);

$$f_{ds} = \pm \frac{\left|\vec{V}_{rel}\right| f_c}{C} \qquad (2.6)$$

f_{ds} = Doppler shift frequency, $\vec{V}_{rel}$ = relative velocity of the satellite and the Earth's terminal sometimes called relative radial velocity between the satellite and the Earth's, f_c = Carrier frequency and C = Velocity of electromagnetic wave

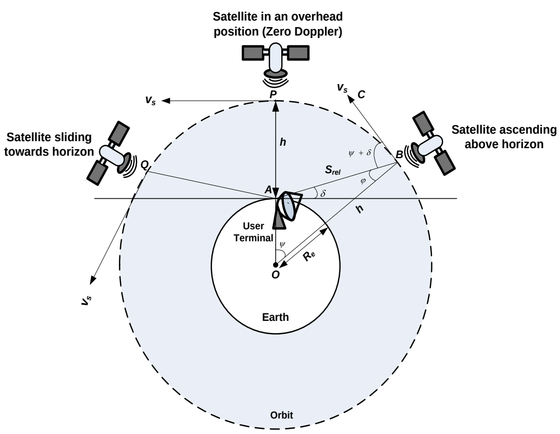

Figure 2.2: Geometrical Representation of Satellite Orbital Dynamics

The ambiguous sign in equation (2.6) explains the scenarios where the satellite is either ascending towards horizon or receding below it with respect to the position of the Earth's terminal. Figure 2.2 shown below is the modification of figure 2.1, showing respectively the elevation angle (δ), satellite coverage angle (ψ) and beam-width (tilt or nadir angle) of the satellite (φ). Figure 2.2 was used to develop the appropriate mathematical representation of Doppler shift as the satellite sweeps over the orbit in an elliptical path.

Applying sine rule to Δ AOB, (δ) and (φ) are related to Earth radius (R_e) and height of the orbit (h) by the following equation;

$$\frac{\sin(90^0 + \delta)}{R_e + h} = \frac{\sin\varphi}{R_e} \qquad (2.7)$$

Hence, $$\sin\varphi = \frac{R_e \cos\delta}{R_e + h} \qquad (2.8)$$

Similarly, $$\varphi = 90^0 - (\psi + \delta) \qquad (2.9)$$

Using equations (2.8) and (2.9), the satellite coverage angle ψ will be;

$$\psi = \cos^{-1}\left(\frac{R_e \cos\delta}{(R_e + h)}\right) - \delta \qquad (2.10)$$

Where $(\psi + \delta)$ is the angle between the satellite and the line joining it with the Earth's terminal.

To compute the relative velocity between the satellite and the Earth's terminal, a radial vector component of the satellite velocity ($\vec{V}_{rel}$) in the direction relative to the Earth's terminal is obtain by the transformation of satellite velocity V_s along line AB of Figure 2.2 (lining joining the satellite and Earth's terminal).

$$\vec{V}_{rel} = V_S \cos(\psi + \delta) \qquad (2.11)$$

Where V_S is the satellite orbital velocity. Using equation (2.10), equation (2.11) becomes;

$$\vec{V}_{rel} = V_S \sin\varphi \qquad (2.12)$$

Substituting equation (2.8) in equation (2.12), the satellite radial velocity relative to the Earth's terminal becomes;

$$\vec{V}_{rel} = \frac{V_S R_e \cos\delta}{R_e + h} \qquad (2.13)$$

Substituting equation (2.13) in equation (2.6), the Doppler shift frequency is written as;

$$f_{ds} = \frac{V_S R_e f_c \cos\delta}{C\left(R_e + h\right)} \qquad (2.14)$$

It can be seen from equation (2.14) that the Doppler shift is zero when the grazing or elevation angle, δ is equal to 90^0. In this position, the satellite is directly overhead the Earth's terminal.

The relative distance between the satellite and the Earth's terminal S_{rel} is calculated from the cosine rule using Δ AOB as;

$$S_{rel} = \left[R^2{}_e + \left(R_e + h\right)^2 - 2R_e\left(R_e + h\right)\cos\psi \right]^{1/2} \quad (2.15)$$

Similarly, S_{rel} can be calculated from sine rule as;

$$\frac{S_{rel}}{\sin\psi} = \frac{R_e + h}{\sin(90^0 + \delta)} \quad (2.16)$$

$$\cos\delta = \left(\frac{R_e + h}{S_{rel}}\right)\sin\psi \quad (2.17)$$

Using equation (2.17), equation (2.14) is further expressed as;

$$f_{ds} = \frac{V_S R_e f_c \sin\psi}{CS_{rel}} \quad (2.18)$$

The magnitude of the velocity of satellite in orbit is calculated from law gravitation as (Carassa, 1989 and Valdoni, 1990);

$$V_S = \frac{2\pi\left(R_e + h\right)}{T} \quad (2.19)$$

Where T is the satellite orbital period. The period of the satellite can be calculated from Kepler's equation as (Valdoni, 1990);

$$T = 2\pi\sqrt{\frac{\left(R_e + h\right)^3}{\mu_L}} \quad (2.20)$$

Equation (2.19) can also be written as;

$$V_S = \sqrt{\frac{\mu_L}{\left(R_e + h\right)}} \quad (2.21)$$

Where $\mu_L = 398\,600\,km^3\sec^{-2} = 3.896\times10^{14}\,m^3\sec^{-2}$ (Kepler's constant).

It can be seen from equations (2.15) through (2.21) that the Doppler frequency shift depends on both the relative velocity and distance between the satellite and Earth's terminal, the frequency of the carrier signal, the height of the orbit and the period of satellite on the orbit.

Doppler frequency shift can be expressed in dB using equation (2.18) as follows;

$$\left[f_{ds}\right] = \left[10\log\left(V_S R_e f_c \sin\psi\right)\right] - \left[10\log\left(CS_{rel}\right)\right]\ (dB) \quad (2.22)$$

A. Effect of the Earth's (User) Terminal Location on Doppler Frequency

Doppler frequency varies as the location of the user terminal changes. This effect is analytically analyzed from the geometry of the satellite and user terminal position shown in Figure 2.3.

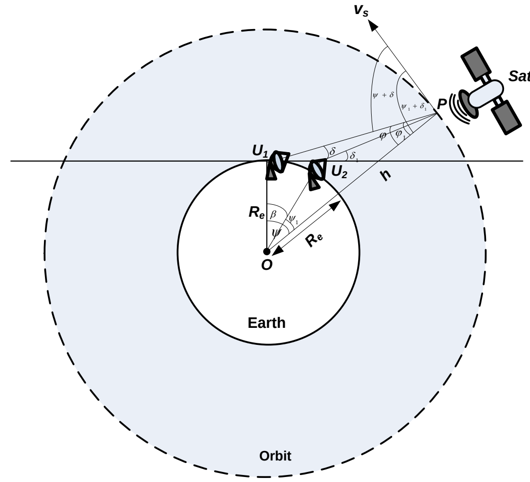

Figure 2.3: Geometry of the Earth's Terminal Location

Suppose the User moves from point U_1 to U_2 through an angle β, the distance traveled is equal to the length of arc $U_1\,U_2$ given by; $\beta = \frac{U_1 U_2}{R_e}$ (β *measured in radian*) (2.23)

Therefore, the new satellite coverage angle becomes; $\psi_1 = \psi - \beta$ (2.24)

III SIMULATION RESULTS

A. Variation of Doppler Frequency with Orbital Height

The fact that Doppler effect is much more pronounced in LEO than in both MEO and GEO is investigated in this section. Using equations (2.15), (2.19) and minimum grazing or elevation angle with its corresponding maximum satellite coverage angle $(\delta_{\min} = 20.07^0\ \ and\ \ \psi_{\max} = 8^0)$ (Carassa, 1989), Doppler frequency shift was plotted against the orbital height for different earth's orbits at different frequency bands (Ku, K and Ka) as shown in Figure 3.1.

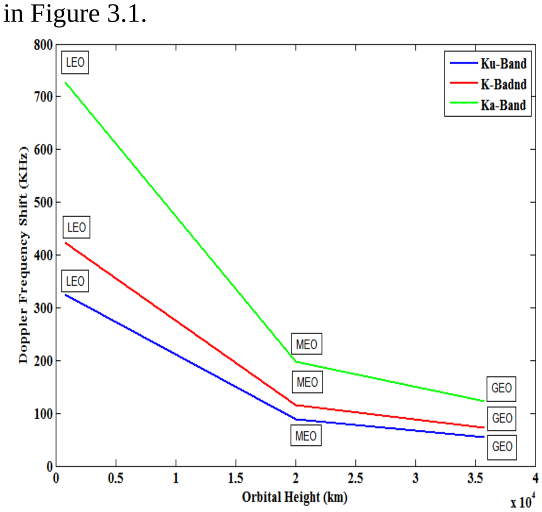

Figure 3.1: Variation of Doppler Frequency with Orbital Height.

As it can be seen from Figure 3.1, there is a sharp decrease in Doppler frequency shift as the height of the orbit increases from LEO through MEO to GEO. This phenomenon is further summarized in Table 3.1.

Table 3.1: Variation of Doppler Frequency with Orbital Height

Type of Orbit	Doppler Frequency (*KHz*)		
	Ku-band	K-band	Ka-band
LEO (780 km)	325.50	423.20	726.90
MEO (20000 km)	88.33	114.80	197.30
GEO (35786 km)	55.26	71.84	123.40

The difference in Doppler Frequencies between LEO and MEO, LEO and GEO are: 237.17 *kHz* and270.24 *kHz* respectively at Ku-band. At K-band, these differences are 308 *kHz* and 351.36 *kHz*. Similarly at Ka-band, the Doppler frequency differences are respectively 529.6 *kHz* and 603.5 k*Hz* . From the foregoing therefore, it can be concluded that Doppler effect is much more severe in LEO than both in MEO and GEO. Having established the effect of Doppler frequency shift in LEO, its variation with respect to the relative distance between satellite and the Earth's terminal for different elevation angles ($0^0 - 90^0$) at Ku, K, and Ka bands was also investigated using equations (2.15) – (2.18); and is depicted in Figure 3.2.

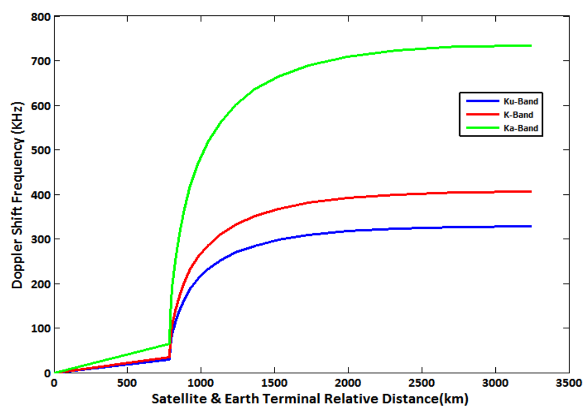

Figure 3.2: Variation of Doppler Frequency with the Satellite and Earth's Relative Distance

It can be seen form Figure 3.2 that between 0-790 km, Doppler frequency increases linearly with the satellite and earth's terminal (user terminal) relative distance. However, beyond this distance, a positive nonlinear increase in Doppler frequency was observed. This is due to continuous change in azimuth angle of the satellite position with respect to the earth's station. Moreover, the Doppler frequency has the highest value of 325.485 kHz at Ku-band, 401.09 kHz at K-band and 745.89 kHz at Ka-band.

B. Effect of User Terminal Location on Doppler Frequency

Knowing the new position of the Earth's terminal, the new relative distance between the satellite and the Earth's station can be calculated. Similarly, the deviation in Doppler frequency from the initial estimated value can also be determined. Equations (2.10), (2.15), (2.18), (2.23) and (2.24) were used to graphically (see Figure 3.3) show the effect of the location of earth's terminal on Doppler frequency.

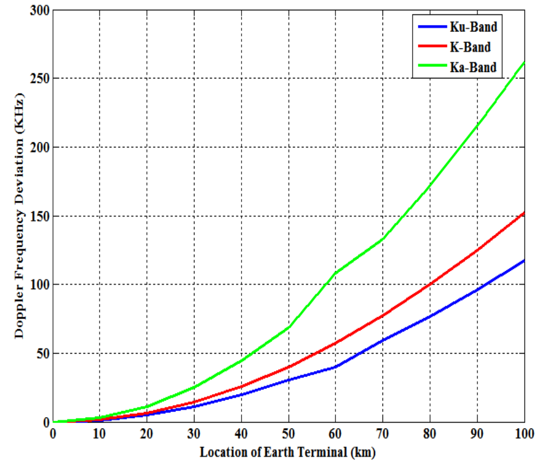

Figure: 3.3: Variation of Doppler Frequency with Earth's Terminal Location

Figure 3.3 shows that the Doppler effect increases with the increase in distance from the initial location of the user terminal. 0 km -100 km was assumed to be the range of distance of user terminal from his the initial to final locations inclusive.

CONCLUSION

The mathematical model of Doppler frequency shift in Low earth orbit (LEO) at Ku, K and Ka was developed and presented. Doppler frequency shift effect was investigated at different satellite orbits (LEO, MEO and GEO). The results obtained show that at maximum satellite converge angle and central frequencies for Ku, K and Ka bands, the Doppler frequencies for LEO (780 km) are: 325.50 kHz, 423.20 kHz and 726.90 kHz; for MEO (20000 km) we have 88.33 kHz, 114.80 kHz and 197.30 kHz; while GEO (35786 km) stood at 55.26 kHz, 71.84 kHz and 123.40 kHz . Variation of Doppler frequency shift with respect to the latitude (location) of the earth's terminal relative to the satellite motion was also studied. A typical earth terminal location in the range of 0 km – 100 km was selected for the study; from which it was verified that effect of Doppler shift in LEO increased as the distance from the initial location of the user terminal increased. These analyses confirm that Doppler effect is more pronounced in LEO than in MEO and GEO.

References

[1] Qingchong, L. (1999): Doppler measurement and compensation in mobile satellite communications systems. In Military Communications Conference Proceedings, 1999. MILCOM 1999. IEEE (Vol. 1, pp. 316-320).

[2] Kausik M., Bera. R. N., and Sil, S. (2007): An Estimation of Doppler Frequency due to Moving Object Like Rain and Others to find its Remedial Measures from a Microwave Communication System. Proc. NCDCTC'07.

[3] Jamalipour, A. (1997): Low earth orbital satellites for personal communication networks. Artech House, Inc..pp51-52 Levanon N. (1988). Radar principles. New York, Wiley- Interscience, p. 1

[4] Gataullin, Y., Dikanskis, D., Häusler, M., Wiedemann, K., Albrecht N., Engelhardt, W., and Strauss, G. (2010): Selection of functional operational requirements for the new

13-M DLR Ka band ground station in Weilheim. International Workshop on Tracking, Telemetry and Command Systems for Space Applications.
[5] Naser Ayat and Mohamad mehdipour (2001): Accurate Doppler Prediction Scheme for Satellite Orbits. Computer engineering group, Payam Noor University
[6] You, M. H., Lee, S. P., and Han, Y. (2000): Adaptive Compensation Method Using the Prediction Algorithm for the Doppler Frequency Shift in the LEO Orbit Mobile Satellite Communication System. ETRI journal, 22(4), 32-39.
[7] Carassa, F. (1989): Application of millimeter waves to satellite systems. Alta Frequenza, pp58, 485-489
[8] Valdoni, F., Ruggieri, M., Vatalaro, F., and Paraboni, A. (1990). A new millimetre wave satellite system for land mobile communications. European Transactions on Telecommunications, 1(5), 533-544.
[9] Raymond P. L., (1996). Communications by means of low Earth orbiting satellites. Modern Radio Science, 133-149.